Dynamical Casimir-Polder Effect with Rydberg Atoms

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**The Dynamical Casimir Effect (DCE)**

Emission of pairs of *real* photons from the vacuum when one of the boundaries of the cavity is set in motion with nonuniform acceleration (parametric amplification)

The most favorable situation is under resonance condition.

Resonance condition: \[ \omega_{wall} = 2\omega_{cavity} \]

Number of emitted photons: \[ n(t) = \langle a^+(t)a(t) \rangle \approx \sinh^2(\omega_{wall}\kappa t) \]
Even under resonance condition, the number of emitted photons is very small. Oscillation frequencies of the order of 10 GHz or more are required in order to measure the DCE. This makes very difficult observing this effect. Such high oscillation frequencies cannot be obtained with a mechanical motion of the cavity wall.

**Dynamical mirror:**

Effective oscillation frequencies around 10 GHz have been obtained with the dynamical mirror. The experiment is in the making in Padova (Italy).

Detection of real photons emitted by DCE through excitation of Rydberg atoms has been recently proposed in the literature.

The number of excited atoms, under realistic experimental conditions, is however small, due to the small number of emitted photons. The DCE has not been yet observed with this experimental setup.


Observation of the DCE or analogous phenomena:
- superconducting circuits by modulating the inductance of a quantum interference device;
- Josephson metamaterial embedded in a microwave cavity;
- Bose-Einstein condensates.

Dynamical Casimir-Polder effect in an atom-mirror system:

It occurs when one or more physical parameters of the atom-wall system are changed non-adiabatically.

Examples:

Sudden change of the atomic transition frequency (e.g. by Stark shift of the atomic levels) or the transition dipole moment.

Sudden change of mirror’s parameters, for example dielectric properties or position.

Optomechanical coupling between an oscillating mirror and (Rydberg) atoms via near-zone Casimir-Polder coupling.
Hamiltonian of a two-level atom interacting with the e.m. radiation field (for \( t < 0 \))

\[
H = H_{\text{field}} + \hbar \omega_0 S_z - \mu \cdot E(r_A)
\]

Fully dressed state at \( t = 0 \) (in the presence of the plate):

\[
|g\rangle_d = |\downarrow_A, \{0_{kj}\}\rangle - i \frac{\sqrt{2\pi c}}{\hbar V} \sum_{kj} \frac{\sqrt{\omega}}{\omega + \omega_0} \left( \frac{\mu \cdot f_{kj}(r_A)}{\omega + \omega_0} \right) |l_{kj}, \uparrow_A\rangle
\]
$t = 0$: sudden change of the atomic transition frequency

$$\omega_0 \rightarrow \omega_0' = \omega_0 + \Delta\omega \quad \Rightarrow H \rightarrow H'$$

(example: Stark shift of the atomic levels by rapidly switching on an external electric field acting on the atom)

The state $|g\rangle_d$ is an eigenstate of the “old” Hamiltonian $H$, but it is not an eigenstate of the “new” Hamiltonian $H'$: for $t > 0$ it is a “partially dressed” state $\rightarrow$ time evolution

Assumption: same state of the system immediately after the switching on of the electric field (i.e., the switching on timescale must be smaller than $\omega_0^{-1}$, otherwise the atom would adiabatically follow the change)
Dynamical Casimir-Polder force for the initial partially dressed state (perfectly conducting mirror)

in the case of the partially dressed state, the force is **not** zero at \( t = 0 \)

The force oscillates in time from attractive to repulsive

- red line: initial partially dressed state
- blue line: initial bare state

\[ t < 2d \quad (c=1) \]
\[ d = 10 \]
\[ \omega_0 = 1 \]
\[ \omega_0' = 2 \]
The asymptotic value of the force is the same in the two cases.

Red line: initial partially dressed state
Blue line: initial bare state

$t > 2d \quad (c=1)$
$d = 10$
$\omega_0 = 1$
$\omega_0' = 2$

The time evolution of the interaction energy depends also on the optical properties of the surface, for example from the dispersion relation of surface plasmon polaritons.

Possibility of detecting a dynamical Casimir-Polder effect through an optomechanical coupling between an oscillating mirror and a gas of Rydberg atoms trapped near the mirror, yielding excitation of the atoms.

The optomechanical coupling occurs through the nonretarded Casimir-Polder interaction (near zone).

This effect is conceptually different from the DCE, because the near field is involved, and it is not related to absorption of the real photons emitted by the oscillating mirror.

The number of excited atoms is significant, hopefully yielding possibility to observe the dynamical effect.
A dynamical Casimir-Polder effect through quantum optomechanical coupling with Rydberg atoms

Static ideal conducting wall

Nonretarded atom-surface Casimir-Polder potential (temperature \( T=0 \)):

\[
V(z) = -\frac{\langle d_x^2 \rangle + \langle d_y^2 \rangle + 2\langle d_z^2 \rangle}{16z^3} \\
= -\frac{1}{16} \frac{\Gamma_{ij} \langle d_i d_j \rangle}{z^3}
\]

\[\Gamma_{ij} = \text{diag}(1,1,2)\]

For the atom-wall distances we shall consider, the real-material corrections can be neglected (\( z \gg \lambda_p \), where \( \lambda_p \) is the plasma wavelength of the wall).

This interaction energy arises from the interaction of the atomic dipole with the vacuum fluctuating field, modified by the presence of the conducting wall.
The atom-wall Casimir-Polder interaction has several regimes according to the atom-wall distance.

- At distances comparable with the plasma wavelength $\lambda_p$ of the wall: the behaviour of the interaction strongly depends on the surface dielectric properties.

- $\lambda_p \ll d \ll c/\omega_0$, van der Waals (nonretarded): $\sim 1/d^3$

- $c/\omega_0 \ll d \ll \lambda_T$, Casimir-Polder (retarded): $\sim 1/d^4$

- $d \gg \lambda_T$, (thermal): $\sim 1/d^3$
Semiclassical model in terms of the image dipole:

In view of the extension to the dynamical case, \( V(z) \) can be seen as the interaction of the atom with an “effective” field due to the image dipole at a distance \( 2z \).

Coupling term: \[ H_I = -\frac{1}{2} \mathbf{d} \cdot \mathbf{E}(\mathbf{r}) \]

Effective field acting on the atom \( \mathbf{r} = (0, 0, z) \) is the position of the atom.

Effective field:
\[
E_i(\mathbf{r}) = \frac{1}{8} \frac{\Gamma_{ij}d_j}{z^3}
\]
Semiclassical effective interaction Hamiltonian

\[ H_I = -d_i \left( \frac{1}{16} \frac{\Gamma_{ij} d_j}{z^3} \right) \]

effective field acting on the atom (due to the image dipole)

We assume that the effective field on the atom, and the time-dependent Casimir-Polder interaction, instantaneously follows the mirror’s oscillation, consistently with the near zone approximation, where electrostatic interactions are relevant.

The average of the effective Hamiltonian on the atomic state coincides with the atom-wall interaction energy.
Extension to the case of a harmonically oscillating mirror

Typical parameters

\[ n = 75 \]
\[ z_0 = 20 \, \mu m \]
\[ a/z_0 = 0.1 \]
\[ f = \omega/2\pi = 30 \, \text{GHz} \]

\[ \lambda \sim 1 \text{ cm} \gg z_0 \, (\text{near zone}) \]
The mirror oscillates harmonically around its equilibrium position \( z=0 \).

\[ z(t) = z_0 \left( 1 - \frac{a}{z_0} \sin(\omega t) \right) \]

**Effective atom-wall interaction Hamiltonian**

\[ H = H_s + V_I(t) \]

\[ V_I(t) \approx -d_i d_j \left( \frac{3}{16} \frac{\Gamma_{ij}}{z_0^3} \frac{a}{z_0} \sin(\omega t) \right) \]

(a \ll z_0)
The time-dependent term $V_I(t)$ can induce transitions between atomic levels.

Excitation probability near resonance \( (\omega = \omega_0) \)

\[
P_e(z_0, t) \approx \frac{9}{2^{10} \hbar^2 \left( \frac{a}{z_0} \right)^2} \left| \frac{\Gamma_{ij} (d_i d_j)^{eg}}{z_0^6} \right|^2 t^2
\]

For a generic (non-harmonic) periodic mirror oscillation \( z(t) = z_0 \left[ 1 - \frac{a}{z_0} f(t) \right] \)

\[
P_e(z_0, t) \approx \frac{9}{2^9 \pi \hbar^2 \left( \frac{a}{z_0} \right)^2} \left| \frac{\Gamma_{ij} (d_i d_j)^{eg}}{z_0^6} \right|^2 \left[ \int_{-\infty}^{\infty} d\omega g(\omega) \frac{e^{-i(\omega - \omega_0)t} - 1}{\omega - \omega_0} \right]^2
\]

Fourier transform of \( f(t) \)
For a Rydberg atom: \( \Gamma_{ij} \left( d_i d_j \right)^{eg} \sim e^2 a_0^2 n^4 \)

\((n \text{ is the initial principal quantum number})\)

\[
P_e(z_0, t) \approx \left( 3 \cdot 10^{-19} \text{ cm}^6 \text{s}^{-2} \right) \frac{a^2}{z_0^8} n^8 t^2
\]

For the following parameters: \( n = 75 \rightarrow n' = 77, z_0 = 20 \mu\text{m}, a/z_0 = 0.1, t = 2 \mu\text{s}, \) the excitation probability is around 20%.

The atomic excitation is a signature of the dynamical Casimir-Polder effect.
Possible experimental setup

Average atom-mirror distance \( (z_0) \): \( 2 \cdot 10^{-3} \) cm

Oscillation amplitude (= thickness of the semiconductor layer of the dynamical mirror): \( 2 \cdot 10^{-4} \) cm

Size of the trap:
Diameter \( (d) \): \( 2 \cdot 10^{-3} \) cm
Length \( (h) \): \( 5 \cdot 10^{-2} \) cm

Number of Rydberg atoms: \( 10^3 \)

Initial principal quantum number: 75

Mirror oscillation frequency, equal to the \( n=75 \to n=77 \) transition frequency (resonance condition): 30 GHz

(for typical parameters of a suitable Rydberg atoms trap, see for example: A. Reinhard et al, Phys. Rev. Lett. 100, 233201 (2008) )
For sufficiently low atomic density, interatomic interactions can be neglected, and the number of excited atoms is (atoms are treated independently)

\[ N_e(t) = \int_0^\infty dz \, \rho(z) P_e(z,t) \]

atomic linear density along \( z \)

First approximation: parabolic gas profile in the trap

\[ \rho(z) = \frac{3N}{4R_z^3} \left[ R_z^2 - (z - z_c)^2 \right] \]

\[ N = \text{number of atoms} \]
\[ z_c = \text{trap center} \]
\[ 2R_z = \text{width of the profile along } z \]
\[ R_z < z_c \]

The number of excited atoms at time \( t \) is:

\[ N_e(t) \approx \left(10^{-20} \text{ cm}^6 \text{ s}^{-2}\right) \frac{3 + 42(z_c/R_z)^2 + 35(z_c/R_z)^4}{\left((z_c/R_z)^2 - 1\right)^6} NR_z^{-8} a^2 n^8 t^2 \]
For $n = 75$, $z_c = 2 \cdot 10^{-3}$ cm, $a = 2 \cdot 10^{-4}$ cm, $R_z = 10^{-3}$ cm, $N = 10^3$,

$$N_e(t) \approx \left(4 \cdot 10^{14} \text{s}^{-2}\right)t^2$$

For $t = 0.5$ μs about 100 atoms of the sample of $10^3$ atoms are excited.

This number should be sufficiently high for detecting the dynamical effect.

**Approximations:**
- interatomic interactions can be neglected compared to the atom-mirror interaction (closest atom-atom distance $\sim 10^{-3}$ cm)
- quadrupolar interactions (atom-wall and atom-atom) can be neglected

Dilute gas of Rydberg atoms trapped near the dynamical mirror
Conclusions

Dynamical Casimir-Polder effect through an optomechanical coupling between an oscillating wall (dynamical mirror) and a dilute gas of trapped Rydberg atoms.

The atoms are subjected to an effective time-dependent field due to the oscillation of the mirror and to the dynamical atom-wall Casimir-Polder interaction $\Rightarrow$ excitation of the atoms.

The near field (nonretarded) is involved in the atomic excitation.

Under realistic experimental conditions, the number of excited atoms seems sufficiently high to allow detection of the dynamical effect.